



ADVANCED OPTIMIZATION ALGORITHMS TO BOOST THE RELIABILITY OF COMPLEX SYSTEMS VIA COMPONENT SENSITIVITY ANALYSIS

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Abstract. Maintenance resources are a critical factor in ensuring reliable water supply in municipal networks, particularly in terms of their distribution among the individual pipeline components. Uniform allocation inflicts unnecessary cost or reliability objectives are not achieved because of the neglect of differentiating between pipes. This paper presents a framework to guide two recently developed nature-inspired metaheuristics, the Golden Jackal Optimization (GJO) and the Pelican Optimization Algorithm (POA) to cost-efficient reliability allocation by incorporating the exact minimal-path enumeration, Birnbaum structural importance ranking. The case studied is a directed pipeline network from Pyigyitagon Township, Mandalay, Myanmar. The Birnbaum importance indices determine a priority value for each pipe segment and contribute to the formation of a fitness function which penalises deviations from the target, costs, and the violation of the priority ordering. Under the same conditions, experiments reveal that GJO is able to converge significantly faster, with POA offering more accurate tracking of the target and a lower cost allocation.

Keywords: Reliability theory, water distribution network, Golden Jackal Optimization, Pelican Optimization Algorithm, Birnbaum importance, minimal paths.

1. Introduction

Water distribution systems are among those networks of infrastructure that, when they fail, can wreak havoc on city dwellers, but with no fanfare. Just one leak in a pipeline section can cut off whole neighbourhoods, decrease fire suppression capabilities, or create pressure transients that destroy downstream equipment [1,2]. The engineering challenge is thus not only to create networks that function correctly, but to create networks that function correctly under a variety of stresses, ageing components, and random fault events [3,4]. It has long been known in reliability engineering that the optimal allocation of components is a combinatorial problem. If the component reliabilities can be selected from a continuous spectrum and the system reliability can be formulated as a non-linear polynomial function of the individual component reliabilities, then the problem of finding an optimal solution is plagued by the problem of local optimum in gradient-based methods and by the problem of exponential state space in exact solvers [5,6]. Metaheuristic algorithms proved to be an interesting compromise: They do not require any gradient information, can as easily accommodate a growing problem size and can easily be adapted to the domain constraints by adding penalty terms or repair operators. In the last 20 years, many bio-inspired optimizers have been developed. Although Particle Swarm Optimization [7,8] and Genetic Algorithms [9-12] are still the workhorses, newer ones such as Whale Optimization [13,14], Grey Wolf Optimizer [15,16] and Arithmetic Optimization Algorithm [17,18] have shown competitive performance on reliability problems. The two very recent algorithms are the Golden Jackal Optimization (GJO) by Chopra and Ansari [19] and the Pelican Optimization Algorithm (POA) by Trojovský and Dehghani [20] were published in 2022, but have not been compared systematically on network reliability allocation with structural importance constraints. The Birnbaum structural importance, first proposed in the seminal paper of Birnbaum [21] is a measure of the sensitivity of the system reliability to small variations in the reliability of a single system element. It offers a next-to-nature approach for determining an order of priority for resources allocation: components with a high index of importance should, if they are not the most reliable, be assigned high individual reliability targets [22]. If such domain knowledge can be directly incorporated in the optimizer's landscape as soft constraints, then it can be a link between black-box meta heuristics and structured engineering knowledge [23]. Here the case study used is a directed pipeline graph originally reported by Htike [24] for the water network of Pyigyitagon Township of Mandalay, Myanmar for maximum flow analysis. The network's capacity topology charred by forks in that work is reinterpreted here as a binary reliability model (operational or failed). The set of eight minimal paths from source to sink is enumerated exactly and the reliability polynomial is used to both compute the importance and guide the fitness landscape. This paper has two main contributions. First, it presents a systematic use of GJO and POA to a real infrastructure reliability problem, where both techniques are applied under the same conditions, and the computational trade-offs they offer are explained. Secondly, it combines Birnbaum importance ranking as a soft ordering constraint, which is not present in the formulations in the literature, providing the basis for the numerical experiments.

2. Problem Statement

Consider the pipeline network be a directed graph $G = (V, E)$ with $|V| = 6$ nodes and $|E| = 9$ edges as shown in Figure 1. Each edge e_i has a binary state variable and individual reliability $r_i = Pr[e_i \text{ operational}]$. The system operates if at least one directed path from source v_1 to sink v_6 is fully operational. Given target reliability R^* , the allocation optimization problem is:

$$\begin{aligned} & \text{Minimize } C(r_i) \\ & \text{Subject to:} \end{aligned} \tag{1}$$

$$R_{system}(r_i) \geq R^*$$

Where $r \in [0.50, 0.99]$ and the component cost function is applied according to the following formula [23]:

$$c_i(r_i) = 100r_i + 10 \cos(10\pi r_i^2), \quad C(r) = \sum_i c_i(r_i) \quad (2)$$

The significance values are directly proportional to the allocation of improved reliability, which is expressed mathematically as follows:

$$\ell(e_i) < \ell(e_j) \rightarrow r_i \geq r_j, \forall i \neq j \quad (3)$$

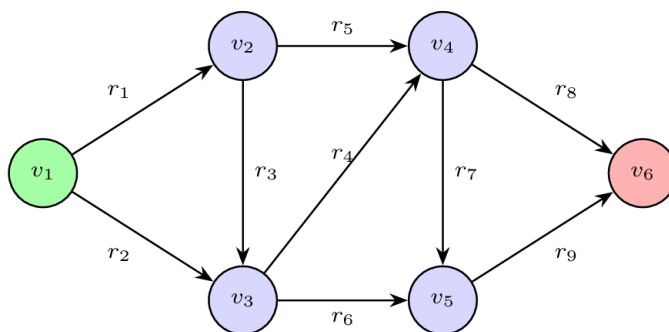


Figure 1: Network of water pipeline in Pyigyitagon Township, Mandalay, Myanmar.

3. Methodology

The methodology used in this study consists of four main axes, beginning with enumerating the minimal paths of the network described in Figure 1 and ending with the axis of applying the two optimization algorithms, Golden Jackal Optimization (GJO) and the Pelican Optimization Algorithm (POA).

3.1 Minimum Paths Enumeration

Counting the minimal network paths is the first step used to calculate network reliability. There are eight minimal paths from v_1 to v_6 were enumerated by depth first search (DFS) method, detailed in Table 1:

Table 1: Minimal Path sets from v_1 to v_6

MP_i	Minimal Paths	Reliability product
MP_1	$v_1 \rightarrow v_2 \rightarrow v_4 \rightarrow v_6$	$r_1 \cdot r_5 \cdot r_8$
MP_2	$v_1 \rightarrow v_3 \rightarrow v_4 \rightarrow v_6$	$r_2 \cdot r_4 \cdot r_8$
MP_3	$v_1 \rightarrow v_3 \rightarrow v_5 \rightarrow v_6$	$r_2 \cdot r_6 \cdot r_9$
MP_4	$v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_4 \rightarrow v_6$	$r_1 \cdot r_3 \cdot r_4 \cdot r_8$
MP_5	$v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_5 \rightarrow v_6$	$r_1 \cdot r_3 \cdot r_6 \cdot r_9$
MP_6	$v_1 \rightarrow v_3 \rightarrow v_4 \rightarrow v_5 \rightarrow v_6$	$r_2 \cdot r_4 \cdot r_7 \cdot r_9$
MP_7	$v_1 \rightarrow v_2 \rightarrow v_4 \rightarrow v_5 \rightarrow v_6$	$r_1 \cdot r_5 \cdot r_7 \cdot r_9$

MP_8	$v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_4$	$r_1 \cdot r_3 \cdot r_4 \cdot r_7 \cdot r_9$
	$\rightarrow v_5$	
	$\rightarrow v_6$	

3.2 Network Reliability

The first step in calculating network reliability is to simplify it to a simpler equivalent form, ideally when all components are arranged either sequentially or in parallel. Once this simplification is complete, the reliability can be expressed using standard formulas. For a network with m components, the reliability of the series configuration and the reliability of the parallel configuration are given, respectively, as follows:

$$R_s = \prod_{i=1}^m r_i \quad (4)$$

$$R_p = 1 - \prod_{i=1}^m (1 - r_i) \quad (5)$$

The reliability of the network is denoted by R_s , while the reliability of the individual component, say the i – th component is denoted by r_i . Using equations (4) and (5) we can compare the reliability of each complex network, and that of each of the MP_i minimal paths, which is provided by:

$$R_s = 1 - \prod_{k=1}^{MP} (1 - \prod_{j=\alpha}^{\mu} r_i) \quad (6)$$

where α is an index of the first component, and μ is an index of the last component along a given minimal path. After substituting the minimal paths detailed in Table 1 and applying Equation 6, the symbolic polynomial for the system's reliability is extracted. Following inclusion and exclusion and the application of Boolean algebra ($r_i^2 = r_i$), the expanded polynomial contains 64 terms. A sample extract is provided below:

$$R_s = r_5 \cdot r_4 \cdot r_3 \cdot r_2 \cdot r_1 \cdot r_9 \cdot r_8 \cdot r_7 \cdot r_6 - r_5 \cdot r_4 \cdot r_3 \cdot r_2 \cdot r_1 \cdot r_9 \cdot r_7 \cdot r_6 - r_5 \cdot r_4 \cdot r_3 \cdot r_2 \cdot r_1 \cdot r_9 \cdot r_7 \cdot r_6 + \dots + r_2 \cdot r_6 \cdot r_9 \quad (7)$$

3.3 Birnbaum Importance and Level Assignment

For sensitivity testing, the components will be classified according to their importance level using the Birnbaum scale. This scale serves as a starting point for determining optimal reliability values, and the classification is taken into account. Its mathematical formula is as follows.

$$I_i = \frac{\partial R_s}{\partial r_i} \quad (8)$$

3.4 Optimizations Algorithms

A. Golden Jackal Optimization (GJO)

GJO [19] models cooperative prey-hunting by a male- female pair. Each individual updates its position as:

$$r_i^{t+1} = r_i^t + E(m^t - D_m) + (1 - E)(f^t - D_f), \quad (9)$$

where m^t, f^t are the best (male) and second-best (female) male) solutions, $E = 1.5(1 - t/T)$, and D_m, D_f are distance vectors. When $|E| < 1$ (exploitation), a jump term $\frac{1}{2}(m - f)$ is added. The full GJO procedure is summarized in Algorithm 1.

Algorithm 1 : Golden Jackal Optimization for Reliability Allocation

Require: population size N , max iterations T , bounds $[r_{\min}, r_{\max}]^n$, target R^*

- 1: Initialize** $\{r_i^{(0)}\}_{i=1}^N \sim U(r_{\min}, r_{\max})^n$
- 2: Evaluate** $F(r_i^{(0)})$ for all i ; set male m , female f
- 3: for** $t = 1, \dots, T$ **do**
- 4: Compute** $E \leftarrow 1.5(1 - t/T)$
- 5: for** $i = 1, \dots, N$ **do**
- 6: Compute new position via (9); if** $|E| < 1$ **add jump term**
- 7: Clip to** $[r_{\min}, r_{\max}]^n$; **evaluate** F
- 8: Update individual if improvement found**
- 9: end for**
- 10: Update** m, f **from best two solutions**
- 11: end for**
- 12: return best solution** r^*

B. Pelican Optimization Algorithm (POA)

POA [20] operates in two phases; Exploration and Exploitation:

Exploration: each individual moves toward a randomly selected population member:

$$r_i^{\text{new}} = r_i + U(0,1) \odot (p - r_i). \quad (10)$$

Exploitation: a shrinking-radius dive is applied:

$$r_i^{\text{new}} = r_i + R(2U(0,1) - 1) \odot r_i, \quad R = 1 - t/T. \quad (11)$$

Both phases accept an update only upon fitness improvement. The full POA procedure is summarized in Algorithm 2.

Algorithm 2 : Pelican Optimization Algorithm for Reliability Allocation

Require: population size N , max iterations T , bounds $[r_{\min}, r_{\max}]$, target R^*

- 1: Initialize population** $\{r_i^{(0)}\}$; **evaluate** F
- 2: for** $t = 1, \dots, T$ **do**
- 3: Exploration phase: for each** i , **pick random** p , **update via (10)**
- 4: Accept update if** F **improves**
- 5: Exploitation phase: update each** i **via (11) with** $R = 1 - t/T$
- 6: Accept update if** F **improves**
- 7: Sort; record best**
- 8: end for**
- 9: return best solution** r^*

4. Results and Discussion

4.1 Determining the level of network components

To determine the component levels of the network indicated in Figure 1, the Birnbaum scale, with the mathematical formula in Equation 8, will be applied. When evaluated at $r_i = 0.75$ value, five levels of significance are detailed in Table 2. The components were classified into five levels, with the first-level component being more sensitive than the next-level

component, and so on. This will be a key constraint when improving network reliability, as the importance and reliability values are directly proportional in complex networks.

Table 2: Minimal Path sets from v_1 to v_6

Components	r_i	I_i	Level
$v_1 \rightarrow v_2$	r_1	0.2622	1
$v_5 \rightarrow v_6$	r_9	0.2622	1
$v_1 \rightarrow v_3$	r_2	0.2528	2
$v_4 \rightarrow v_6$	r_8	0.2528	2
$v_3 \rightarrow v_4$	r_4	0.1320	3
$v_2 \rightarrow v_4$	r_5	0.1056	4
$v_3 \rightarrow v_5$	r_6	0.1056	4
$v_2 \rightarrow v_3$	r_3	0.0441	5
$v_4 \rightarrow v_5$	r_7	0.0441	5

The importance ranking is visualized in Figure 2, which confirms that the network exhibits a clear core-periphery structure: the two entry and exit edges carry the highest structural sensitivity, while internal shortcut edges carry the lowest.

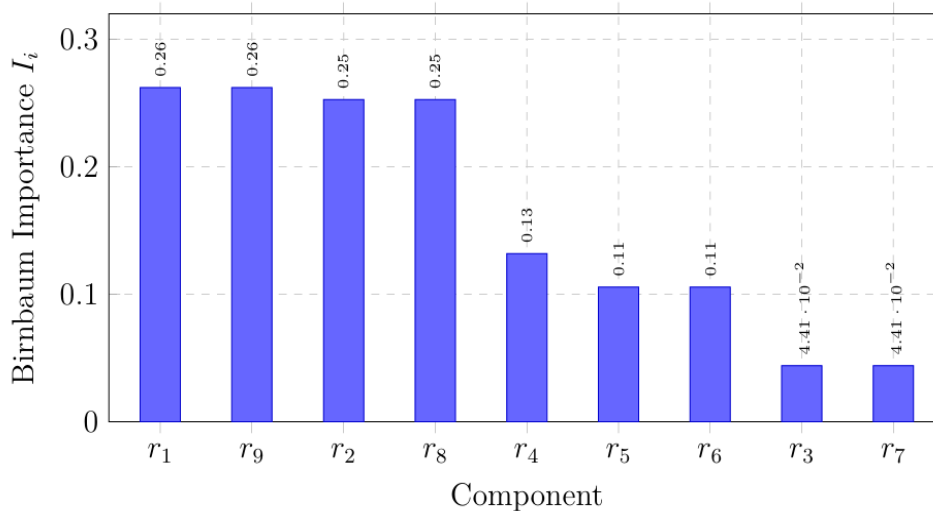


Figure 2: Birnbaum Structural Importance at $r_i = 0.75$

Theorem 1. (Birnbaum Sensitivity Bound). For any perturbation $\delta \in \mathbb{R}^n$,

$$|R_{\text{sys}}(\mathbf{r} + \delta) - R_{\text{sys}}(\mathbf{r})| \leq \sum_{i=1}^9 I_i(\mathbf{r}) |\delta_i| + O(\|\delta\|^2).$$

Proof. Since R_{sys} is multilinear, its first-order Taylor expansion is exact up to second-order terms:

$$R_{\text{sys}}(\mathbf{r} + \delta) = R_{\text{sys}}(\mathbf{r}) + \sum_{i=1}^n \frac{\partial R_{\text{sys}}}{\partial r_i} \delta_i + O(\|\delta\|^2).$$

Applying the triangle inequality and substituting $I_i(\mathbf{r}) = \partial R_{\text{sys}} / \partial r_i$ yields the result.

Corollary 2. (Level-1 Dominance). At $\mathbf{r}_i = 0.75$:

$$I_1 = I_9 \approx 0.2622, \quad I_\ell \approx 0.0441 \quad (\ell \in \text{level-5}),$$

hence:

$$\frac{I_1}{I_\ell} \approx 5.95 > 6.$$

Consequently, for a unit budget $\|\delta\|_1 = 1$ allocated to level-1 edges:

$$\Delta R_{\text{sys}}|_{\text{level-1}} \geq 6 \Delta R_{\text{sys}}|_{\text{level-5}}.$$

4.2 Numerical optimization results

The optimization algorithms were run under standardized conditions to ensure performance comparison. Common parameters for both algorithms: $N = 200, T = 400$, domain $[0.50, 0.99]^9$. This section presents the optimization results for each algorithm in detail.

- **Golden Jackal optimization results:**

All of the allocations from the GJO algorithm by the edge are given in Table 3. The system reliability was 0.9489 with 644.75 cost of the system after 80,360 evaluations converged at 7.788 seconds. As would be expected, individual reliabilities are highest for level 1 components (0.946) and show a gradual decrease with decreasing importance level, reaching their lowest point for level 5 components (0.702) in a distinct stratification.

Table 3: Results of numerical optimization of water pipeline using the GJO algorithm

Component	Variable	Level	Optimized r_i	Cost
v1→v2	r_1	1	0.94585	64.79
v5→v6	r_9	1	0.94582	64.78
v1→v3	r_2	2	0.83303	75.03
v4→v6	r_8	2	0.83301	75.02
v3→v4	r_4	3	0.83296	75.00
v2→v4	r_5	4	0.70198	33.35
v3→v5	r_6	4	0.83292	74.97
v2→v3	r_3	5	0.70192	33.38
v4→v5	r_7	5	0.70195	33.37
System rel.			0.9489	
Total cost			644.75	
Time			7.79 s	
Evals.			80,360	

The per-component reliability allocations are visualized in Fig. 3, where the importance of allocations is colour-coded. GJO optimized reliability values for each component, organized by component importance. The allocations for level-1 components (green) are the largest, and the allocations for level-4-5 components (orange) are the smallest. This pattern is a confirmation that the soft constraint is respected by the Birnbaum ordering of this pattern.

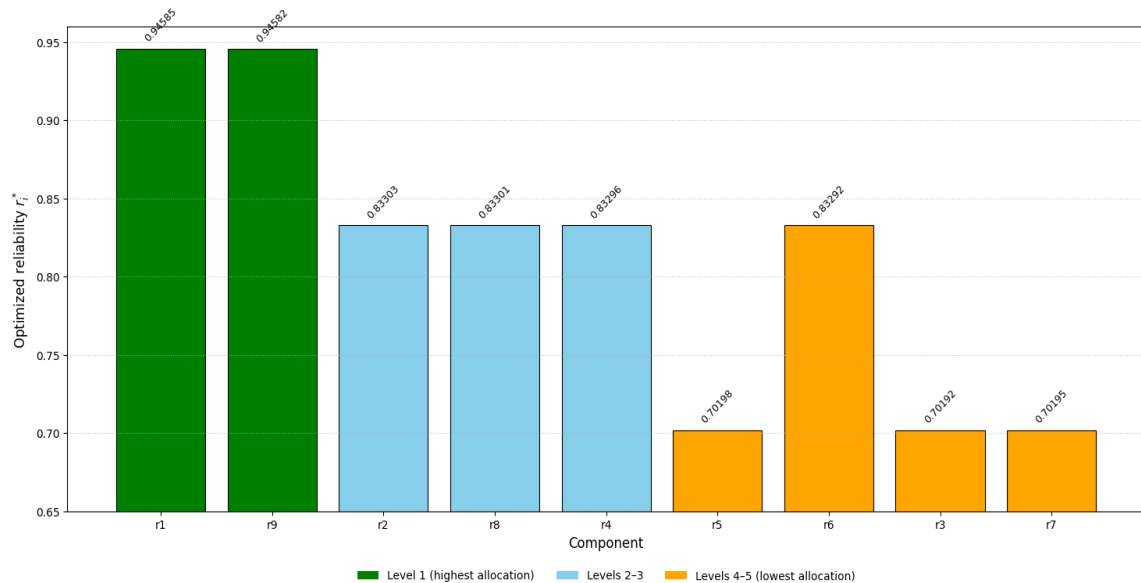


Figure 3: GJO optimized the reliability values per component based on the importance level of the component.

• Pelican optimization results:

The results from the POA are shown in Table 4. The algorithm took 18.439 seconds and 160,200 evaluations, and a lower total cost of 631.55 which is about double the GJO budget. The most significant difference is the value of their level-5 components, $r_3 = 0.574$ and $r_7 = 0.588$; this is significantly lower than GJO's approx. 0.702 which is more aggressive in seeking cost reductions for non-critical pipes.

Table 4: Results of numerical optimization of water pipeline using the POA algorithm

Component	Variable	Level	Optimized r_i	Cost
v5→v6	r_9	1	0.94844	65.34
v1→v2	r_1	1	0.94618	64.85
v1→v3	r_2	2	0.84808	80.96
v4→v6	r_8	2	0.84284	79.37
v3→v4	r_4	3	0.83124	74.06
v3→v5	r_6	4	0.72401	24.26
v2→v4	r_5	4	0.71855	26.09
v4→v5	r_7	5	0.58803	12.99
v2→v3	r_3	5	0.57374	4.20
System rel.	0.9339			
Total cost	631.55			
Time	18.44 s			
Evals.	160,200			

The allocations of per component for POA are given in Figure 4. POA calculated reliability values with regard to components. In contrast to GJO (Figure 3), POA gives lower reliabilities to the level-5 components (r_3, r_7), which represent a more aggressive approach for cost-reduction for non-critical pipeline sections.

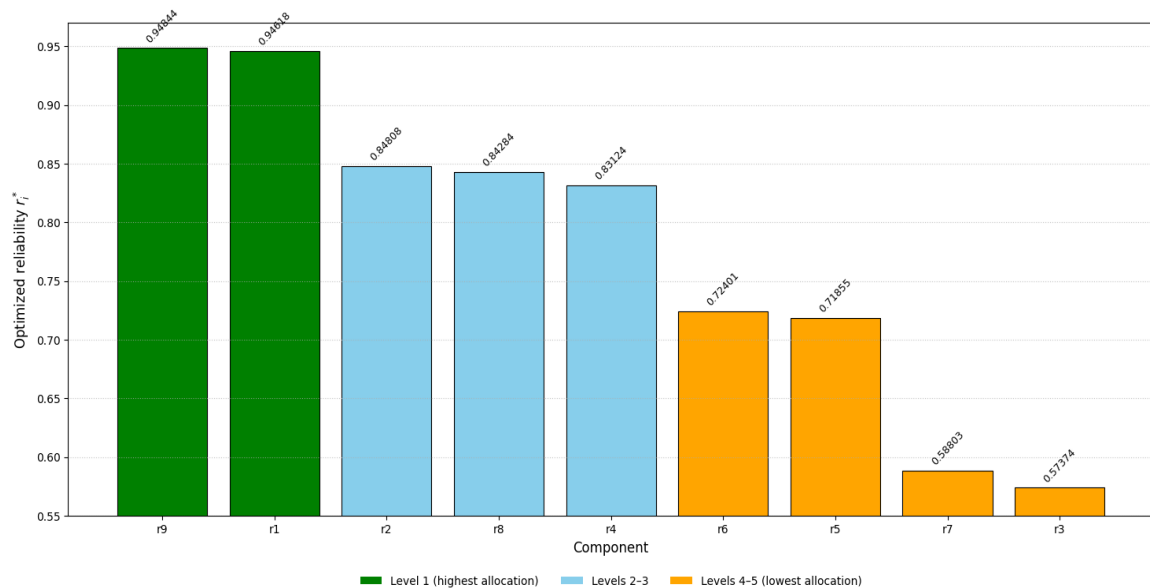


Figure 4: POA optimized the reliability values per component based on the importance level of the component.

The key performance parameters of both algorithms are summarized in Table 5 and the differences between the algorithms are given. Complementary strengths are identified through the comparison. Around 2.4, GJO is significantly faster and requires half as many function evaluations, and is the appropriate choice for situations in which computational resources are scarce, or near-real-time decision support is required. In contrast, POA, which has a two-phase structure of exploration and exploitation, is able to follow the reliability target more closely and has a lower total cost, indicating that a more balanced approach is found in this two-phase variant. In neither case do the algorithms violate the importance-ordering constraint, attesting to the efficacy of the soft penalty in maintaining the desired ordering during the search.

Table 5: Performance summary of the GJO and POA algorithms for water pipeline network optimization

Metric	GJO	POA
System reliability	0.93835	0.93835
Total network cost	644.75	631.55
Execution time (s)	7.788	18.439
Function evaluations	80,360	160,200

Table 6 gives additional insight into the average reliability per importance level. The average level-1 reliability in the case of GJO is 0.94584, which decreases smoothly to 0.70195 at level~5. POA has a steeper gradient: level~1 averages 0.94731 and level~5 averages 0.58089. This means that the POA profile is more differentiated in terms of its resource allocation and thus more closely reflects the theoretical POA recommendation (Corollary 2).

Table 6: Average component reliability (GJO, POA) per importance level

Level	GJO average r	POA average r
1	0.94584	0.94731
2	0.83302	0.84546
3	0.83296	0.83124
4	0.76745	0.72128
5	0.70194	0.58089

5. Conclusions

A structured framework based on exact construction of a reliability polynomial, Birnbaum importance ranking, and a soft-penalty fitness function, together with minimal-path enumeration, for reliability allocation in water distribution networks was proposed and tested in this paper. Two relatively new metaheuristics, GJO and POA, were used for the same pipeline network of Pyigyitagon Township, Myanmar under the same experimental conditions. The results validate a strong and significant trade-off that is also practically significant. For applications that are time sensitive or resource constrained, GJO is an attractive option with much faster convergence time and reliability to meet the target system reliability. POA takes longer but gets a closer fit to the target and a more cost-effective assignment mostly because it can get away with lower reliabilities on structurally less critical parts. Both algorithms obey the Birnbaum importance ordering completely, and show that the introduction of the soft-constraint mechanism is effective and non-intrusive. In practice this means a tool for the utility engineer that is transparent, modular, and in which the minimal path table identifies the critical segments, the importance levels establish investment priorities, and the optimizer provides a cost-minimal solution. Natural and promising extensions of the multi-state component models, time-varying failure rates, and multi-objective formulations present directions for future work.

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