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THE INFLUENCE OF SUBSTRATE AVAILABILITY ON THE MATHEMATICAL MODELING OF FUNGAL GROWTH DYNAMICS

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Abstract: This research aims to develop mathematical models to describe the growth of fungal colonies, focusing on two main factors: the death of fungal threads (hyphae) and the availability of food (substrate). By understanding these mechanisms, we can explain the role of fungi in biological processes such as natural decomposition, in addition to their practical applications in agriculture to combat harmful fungi, and in industry to improve the production of biomaterials. The main objective of this research is to verify the role of the substrate in the fxw model. Their models consist of a system of partial differential equations where the variables refer to the density of fungal threads, the density of tips, and the concentration of the growth-limiting substrate. To analyze this model, we rely on advanced mathematical tools such as stability theory and traveling wave analysis. We also use specialized programs such as **MATLAB** and **Maple** to conduct numerical simulations and verify the accuracy of the results

Keywords: Lateral branching, Tiptip anastomosis, Tip death due to overcrowding

Introduction

This research aims to present a comprehensive mathematical model that explains fungal growth, taking into account the crucial role of available substrate. Mathematical modeling is an indispensable tool for understanding and predicting the behavior of complex biological systems, including microorganisms[8][3]. In the context of fungi, substrate availability directly affects growth rate, biomass density, and morphological patterns. By developing this mathematical model, we can gain a deeper understanding of fungal growth dynamics, optimize the optimal conditions for culturing fungi, and predict their spread in diverse environments[6]. This study is based on a thorough review of the relevant scientific literature and the formulation of an integrated mathematical model based on differential equations. This model will be numerically simulated, and the results will be graphically analyzed. The research will detail the basic components of the model, the parameters used, and the main conclusions drawn from the simulation**.** Based on the partial differential[7][9] equations below, the research will address the following questions[1][4]:

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$\frac{\partial p}{\partial t} = nvs - d(p)$	
$\frac{\partial n}{\partial t} = -\frac{\partial nvs}{\partial x} + \sigma(p,n,s)$	....(1)
$\frac{\partial s}{\partial t} = -kn$	

Where : p is branching, n is tips, and s is substrate

We study a new kind of Lateral branching, Tiptip anastomosis, and Tip death due to overcrowding (**FXWS**) where represents

$\mathbf{F} = \propto_2 \mathbf{p}$	,Where : $\propto_2$ is the number of branches produced per unit length hypha per unit time
$\mathbf{X} = -\beta_3 \mathbf{p}^2$	,Where: β_3 the rate at which overcrowding (density limitation) eliminates branching
$\mathbf{W} = -\beta_1 \mathbf{n}^2$	Where: β_1 is the rate of tip reconnections per unit time

Through the system (1), let us assume $v \neq 0, d(p) = 0,$ we can describe the growth of fungal threads after taking Non-dimensionlision , we get the following system[5]

$\frac{\partial p}{\partial t} = ns$	
$\frac{\partial n}{\partial t} = -\frac{\partial ns}{\partial x} + \alpha p(s-p) - \beta n^2$	....(2)
$\frac{\partial s}{\partial t} = -kn$	

Where $\sigma(p,n,s) = \propto_2 p - \beta_3 p^2 - \beta_1 n^2$, $\alpha = \frac{v\alpha_1}{\gamma^2}, \bar{p} = \frac{\alpha_1}{\beta_3}, \beta = \frac{\beta_1 \alpha_1 \gamma}{v\beta_2}$

Stability of uniform solution

In this part, we illustrate how can lay these parameters to get dimension less

$$ns = 0, \quad \alpha p(s-p) - \beta n^2 = 0, \quad -kn = 0$$

The solution of the system above, we obtain the values of(p,n,s) is$(0,0,s_0), (p_0,0,0)$, where p_0, s_0 is constant. Now, we can take Jacobain for the system (2)

$$J_{(p,n.s)} = \begin{bmatrix} 0 & s & n \\ \alpha s - 2\alpha p & 2\beta n & \alpha p \\ 0 & -k & 0 \end{bmatrix}$$

Thuse

$$J_{(0,0,s_0)} = \begin{bmatrix} 0 & s_0 & 0 \\ \alpha s_0 & 0 & 0 \\ 0 & -k & 0 \end{bmatrix}$$

$\lambda_1 = 0, \quad \lambda_{2,3} = \mp\sqrt{\alpha}\, s_0$ By using$|A - \lambda I| = 0$, We get three eigenvalues of λ:

We can use the determinant and trace of the matrix above to find the steady state : tr(matrix)=-3 λ, and det (matrix) = $\alpha\lambda {s_0}^2 - \lambda^3$

Since we get $\lambda_i = 1,2,3$ and note from above if $\lambda > 0$, we get steady state is stable focus but if $\lambda < 0$ the steady state is saddle point

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$$J_{(p_0,0,0)} = \begin{bmatrix} 0 & 0 & 0 \\ -2\alpha p_0 & 0 & \alpha p_0 \\ 0 & -k & 0 \end{bmatrix}$$

By using$|A - \lambda I| = 0$, We get three eigenvalues of λ: $\lambda_1 = 0$, $\lambda_{2,3} = \mp\sqrt{-\alpha k p_0}$

We can use the determinant and trace of the matrix above to find the steady state : tr(matrix)=-3 λ, and det (matrix) $=-\lambda(\mathrm{k}\alpha\mathrm{p} + \lambda^2)$

note from above if $\lambda > 0$, we get the steady state is saddle point but if $\lambda < 0$ the steady state is un stable focus

Traveling Wave Solution

Let's assume we have three functions, $p(x,t), n(x,t),$ and $s(x,t),$ all of which move at a constant speed (c) in the positive direction of the x-axis. Our goal is to find solutions for these functions in terms of a new variable $(z),$ where $z = x - ct$. This procedure will help us uncover the "traveling wave solution" for the system described in equation (2).To simplify this system[5][1], we can use three ordinary differential equations, as shown below

$\frac{\mathrm{dP}}{\mathrm{dZ}} = \frac{-1}{C} NS$	
$\frac{\mathrm{dN}}{\mathrm{dZ}} = \frac{1}{(S-C)}\left[N^2(\frac{-K}{C} + \beta) + \alpha P(S-P)\right] \quad S \neq C, \quad C \neq 0$	….(3)
$\frac{\mathrm{dS}}{\mathrm{dZ}} = \frac{K}{C} N$	

As previously indicated, we can determine the equilibrium points for system (3). The system consists of two lines of steady states, namely point $(0,0,S)$ and point $(P,0,0)$.To analyze and evaluate the stability of the mathematical model used, we follow a methodology similar to the approaches applied in previous steps, we get tow point(p,n,s)

If $C > 0, \lambda < 0$ $p_0 \leq c, s < c,$ then$(P,0,0),$ is unstaple foucs. But the $(0,0,S)$ is saddle point .

Numerical Solution

Since it is impossible to find an exact analytical solution for system (2), it becomes necessary to resort to numerical methods[1], which can be applied using a function **pdep** code in **MATLAB**:

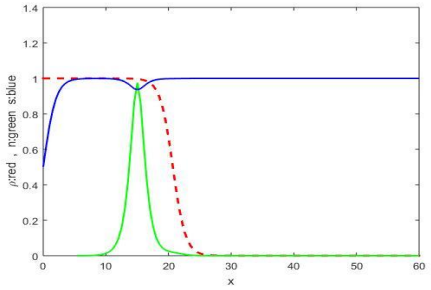

Figure (1): As shown in the Figure, the geometric model for solving system (2) consists of three main elements: the branch $\boldsymbol{p}$: (green), the tips $\mathbf{n}$: (red), and the substrate $\mathbf{s}$: (blue)

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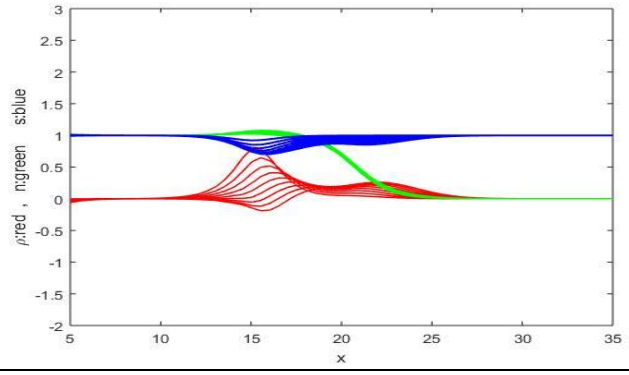
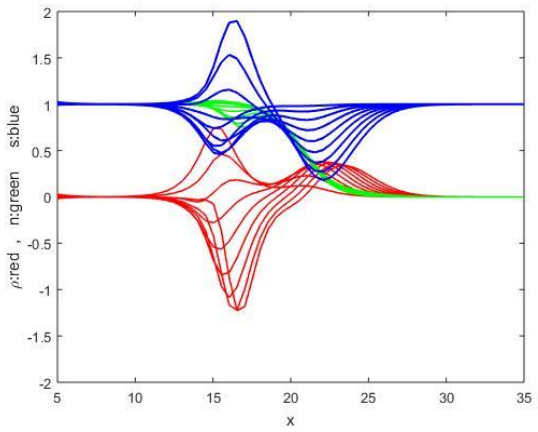
The results show, as expected, that autocatalytic coupling causes an interaction between the $\mathbf{p}, \mathbf{n}$, and $\mathbf{s}$ wave fronts. The validity of this model depends on the parameters $\boldsymbol{\alpha}, \boldsymbol{\beta}, \mathbf{v}, \boldsymbol{d}$, and $\mathbf{k}$ that govern the system (2). In Figure (2), we therefore derived a finite solution based on the parameters $\boldsymbol{\alpha} = \mathbf{0.5}, \boldsymbol{\beta} = \mathbf{0.8}, \mathbf{v} = \mathbf{0.1}$, and $, \boldsymbol{k} = \mathbf{0.3}$. This solution clearly shows the resulting patterns of the substrate, the tip, and the branch, noting that the wave moves in a left-to-right direction:

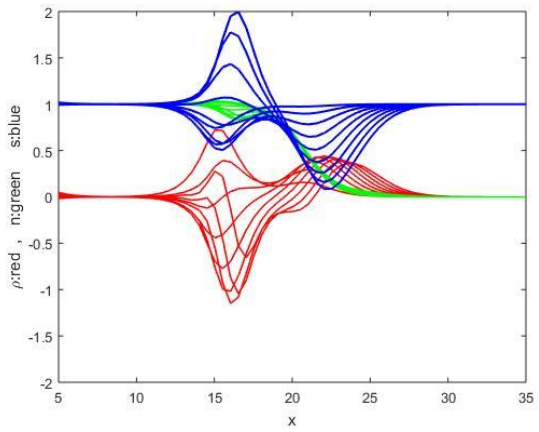


Fig(2): The solution to PDE system (3.33) for FXDS with the parameters $(\boldsymbol{\alpha} = \mathbf{0.5}, \boldsymbol{\beta} = \mathbf{0.8}, \mathbf{v} = \mathbf{0.1}, \boldsymbol{k} = \mathbf{0.3})$ for time $\mathbf{t}$ **[5].**

Now we discuss the effect for all $\boldsymbol{\alpha}, \boldsymbol{\beta}, \mathbf{v}$, and $\mathbf{k}$ of Solution:
The effect for all $\boldsymbol{\alpha}, \boldsymbol{\beta}$ of solution



(a)



(b)

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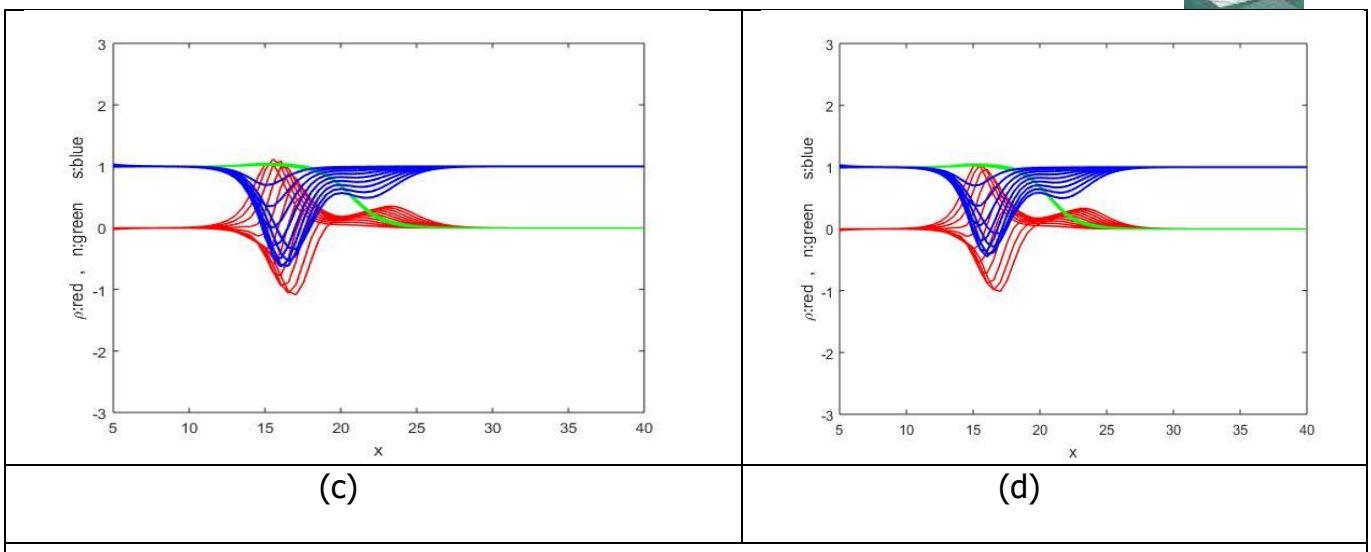
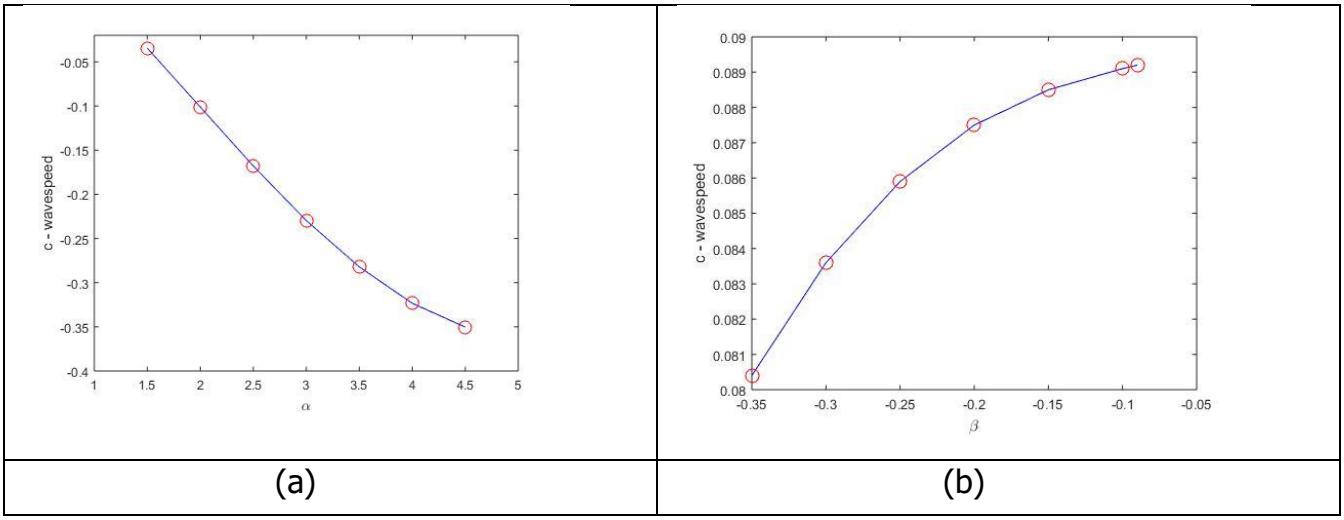


(c) (d)

Fig(3): The solution of PDEs system (3) for FXWS with the parameters $\boldsymbol{\alpha, \beta, v,}$and $\mathbf{k}$. Taking values of the figer(a,b) $\boldsymbol{\beta = 0.8, v = 0.1, k = 1,}$ where $\boldsymbol{\alpha = (1.5,\ 2)}$, but Taking values of fig (c, d) $\boldsymbol{\alpha = 3, v = 0.6, k = 2.5}$, where $\boldsymbol{\beta = (-0.35, -0.25)}$

By using$\boldsymbol{\beta = 0.8, v = 0.1, k = 1,}$ we can obtain the relationship between the traveling wave solution $\boldsymbol{\beta}$ and the values of $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$. We observe that the traveling wave solution $\mathbf{c}$ grows with increasing $\boldsymbol{\alpha}$,and decreasing , $\boldsymbol{\beta}$ where $\boldsymbol{\alpha = 1.5, \alpha = 2}$, and by using $\boldsymbol{\alpha = 3, v = 0.1, k = 1}$ where $\boldsymbol{\beta = (-0.35, -0.25)}$. See the Fig (4)



(a) (b)

Fig(4) In (a)The relationship that exists between ($\boldsymbol{\alpha}$) values and wave speed (c), in(b) The relationship that exists between ($\boldsymbol{\beta}$) values and wave speed (c):

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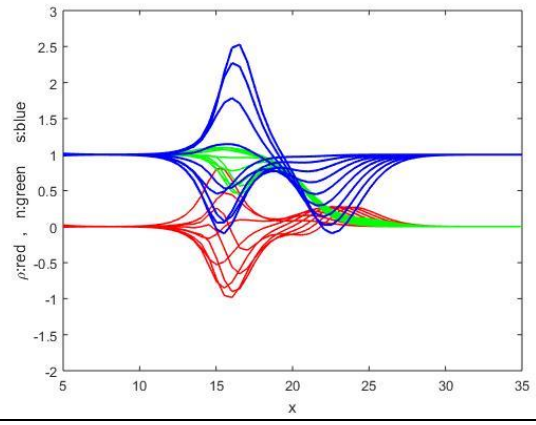
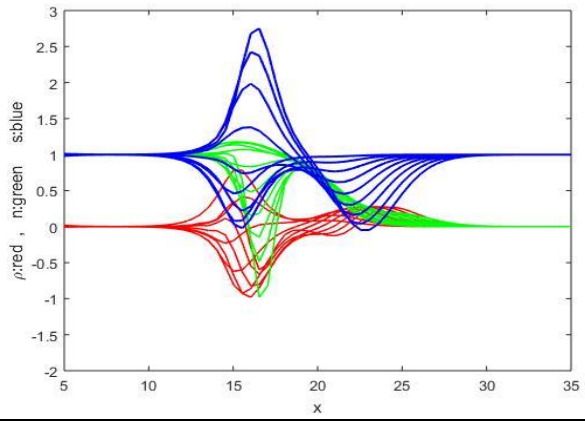
But the effect of v noted:



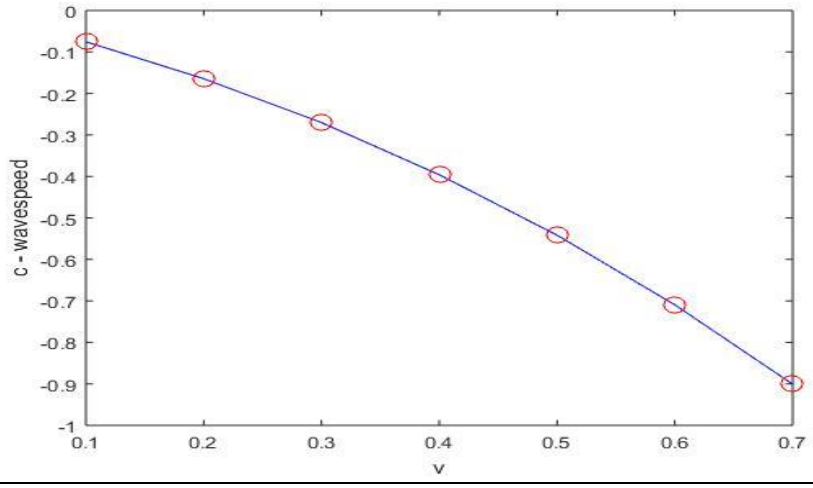
(a)



(b)

Fig(5): The solution of PDEs system (3) for FXWS with the parameters $\boldsymbol{\alpha, \beta, v,}$and $\mathbf{k}$. Taking values of the figer(a, b) in (a) $\boldsymbol{\alpha = 1.5, \beta = 0.3, k = 2,}$ where $\mathbf{v = 0.4,}$ but in b $\mathbf{, v = 0.7.}$

By using $\boldsymbol{\alpha = 1.5, \beta = 0.3, k = 2,}$ we can obtain the relationship between the traveling wave solution $\mathbf{c}$ and t*he values of* $\boldsymbol{v}$*. We observe that the traveling wave solution c grows with increasing* $\boldsymbol{v}$*, where* $\boldsymbol{v = 0.4,\ 0.7}$ *. See the Fig (6)*



Fig(6): In (a)The relationship that exists between ($\mathbf{v}$) values and wave speed (c)

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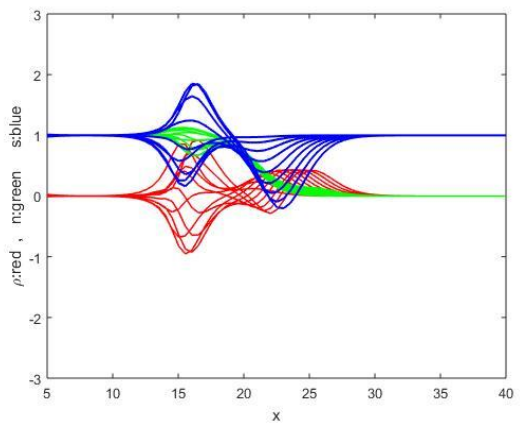
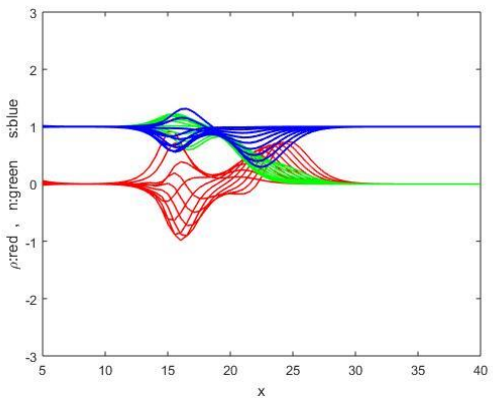
While the effect of k noted:



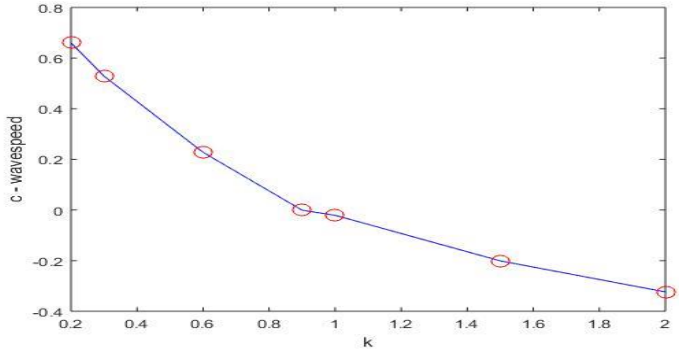
(a)



(b)

Fig(7): The solution of PDEs system (3) for FXWS with the parameters $\boldsymbol{\alpha}, \boldsymbol{\beta}, \mathbf{v}$,and $\mathbf{k}$. Taking values of the figer(a, b) in (a) $\boldsymbol{\alpha} = \mathbf{1.5}, \boldsymbol{\beta} = -\mathbf{0.03}, \mathbf{v} = \mathbf{0.4}$, where $\mathbf{k} = \mathbf{1}$, but in b $\mathbf{k} = \mathbf{0.6}$,.

By using $\boldsymbol{\alpha} = \mathbf{1.5}, \boldsymbol{\beta} = -\mathbf{0.03}, \boldsymbol{v} = \mathbf{0.4}$, we can obtain the relationship between the traveling wave solution $\boldsymbol{c}$ *and the values of* $\boldsymbol{k}$*. We observe that the traveling wave solution c grows with increasing* $\mathbf{k}$*, where* $\mathbf{k} = \mathbf{1}, \mathbf{0.6}$ *. See the Fig (8)*



Fig(8) :The relationship that exists between ($\mathbf{k}$) values and wave speed (c)

Now analyze the results using **MATLAB**

Conclusion

From the previous figures we noticed the effect of the substrate and the parameters α, β, v, and k on the development of fungal networks.

From the previous figures, we observe the following, Figure (a, 4): The relationship between c and α, where the speed of wave c decreases when α is an increasing function. That is, the relationship is inverse. Figure (b, 4): The speed of wave c increases when β is a decreasing function. Also the Figure (6): Shows the relationship between wave speed and v, where the wave speed (i.e., the growth speed) increases when v is decreasing. That is, the relationship between c and v is inverse. While the Figure (8): Shows that the wave speed increases as the value of k decreases.**:**



In contrast, as γ decreases, the growth rate falls while α, and v remain unchanged. As we observe, the colony growth rate remains unaffected by modifications to the anastomosis rate β_3, since α is not dependent on s. The Density accumulation inside would be increased by decreasing β, since $\alpha = \frac{v\alpha_1}{\gamma^2}, \bar{p} = \frac{\alpha_1}{\beta_3}, \beta = \frac{\beta_1\alpha_1\gamma}{v\beta_2}$

Ultimately, the goal of this study is to build a powerful mathematical tool that will allow us to gain a deeper understanding of how fungi live, predict their behavior, and better harness their power for the benefit of humans and the environment

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