Journal of Kyzylorda Scholarly Review

P-ISSN:3006-0265 | E-ISSN:3007-4681. Published by the Open University of Kyzylorda

https://bulletin.ouk.kz/index.php/bulletin/index

An Empirical Application Using Real Data to Compare Two Methods for Estimating and Testing the Parameters of a Poisson Regression Model

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Abstract. Estimation methods are an important cornerstone of regression analysis, and based on an accurate model will result in a well-represented analysis. In this study, a comparison was made between the two methods of maximum likelyhood and the least absolute deviation. Thi Qar for the year 2018. two criteria were used for the differentiation between the models, , the criterion of the coefficient of determination (R^2) and the criterion of the mean squares error (MSE). The maximum likelihood method is better than the less absolute deviation method.

Keywords: Poisson distribution, maximum likelihood, least absolute deviation, coefficient of determination, mean squares error, Wald test.

Introduction

Both the Maximum Likelihood method and the Least Absolute Deviation approach are regarded as key techniques for estimating the parameters of the Poisson regression model. The Maximum Likelihood method is based on maximizing the likelihood function to obtain the parameter estimates of the regression model. Maximum Likelihood estimators possess several asymptotic properties, including that they are asymptotically biased (comparatively biased) and asymptotically efficient (comparatively efficient), and their distribution approaches the normal distribution.[1]

The other method, the Least Absolute Deviation, is based on minimizing the sum of absolute deviations resulting from the difference between the response variable and its Estimated values of the response variable. For the purpose of comparing the two methods, two evaluation criteria were used: the coefficient of determination and the mean squared error. In addition, the significance of the estimated parameters can be assessed using the Wald test.

Poisson Distribution

The Poisson distribution is a discrete statistical distribution that models the frequency of events occurring within a specified interval of time or space.. It is named after the French mathematician Siméon Denis Poisson, who introduced it in

1837 in his work on probability theory. It is suitable when events are rare, independent, and occur at a constant average rate. One of its key properties is that the mean equals the variance. It is widely used due to its simplicity and applicability in fields such as statistics, economics, and education.[2]

If the random variable (X) follows a Poisson distribution, then its probability mass function (p.m.f) is given as follows:

$$\Pr(X = x) = \frac{\theta^x e^{-x}}{x!} \quad , \quad x = 0,1,2,\dots\dots \quad \dots\dots\dots\dots (1)$$

One of the most important properties of the Poisson distribution is that its mean is equal to its variance:

$$\mu x = E(x) = \theta = \text{Mean}$$
$$\text{Variance} = \sigma_x^2 = \text{var}(x) = \theta$$

Methodology

Maximum Likelihood Method

When taking a sample of pairs of observations (x_i, y_i) that are independent of each other, the likelihood function is given by the product of the conditional probability functions for each observation of the response variable (Y), which follows a Poisson distribution conditional on the explanatory variable (X), and it is given as follows: [3][4].

$$P(Y = y_i) = \frac{e^{-\theta_i}\ \theta^{Y_i}}{Y_i!} \quad , \quad i = 1,2,\dots..n \quad \dots\dots\dots..(2)$$

The likelihood function is obtained by maximizing the observed values of the dependent variable distribution (Y_i) as given in the formula above. takes the following form:

$$L(Y_1, Y_2, \dots\dots, Y_n, \theta) = \frac{e^{-\sum_{i=1}^{n}\theta_i}\ \theta^{\sum_{i=1}^{n} Y_i}}{\prod_{i=1}^{n} Y_i!}$$

By taking the natural logarithm of the likelihood function for the above observations, we obtain:

$$\text{LnL}(\underline{Y}_i/X_i, \underline{\beta}) = -\sum_{i=1}^{n}\theta_i + \sum_{i=1}^{n} Y_i\,(\text{Ln}[\theta_i]) - \text{Ln}[\prod_{i=1}^{n} Y_i!] \quad \dots\dots\dots..(3)$$

Based on the fundamental assumptions of the Poisson distribution, which are substituted into the main distribution function, we obtain:

$$\text{LnL}(\underline{Y}_i/X_i, \underline{\beta}) = -\sum_{i=1}^{n} e^{(x_i'\underline{\beta})} + \sum_{i=1}^{n} Y_i\left(\text{Ln}\left[e^{(x_i'\underline{\beta})}\right]\right) - \text{Ln}[\prod_{i=1}^{n} Y_i!] \quad \dots\dots\dots..(4)$$

By differentiating equation (4) with respect to the parameter vector $(\underline{\boldsymbol{\beta}})$ we obtain:

$$\frac{\partial LnL}{\partial \underline{\beta}} = \sum_{i=1}^{n}(Y_i - e^{(x_i'\underline{\beta})})x_i \quad \dots\dots\dots(5)$$

The maximum likelihood estimators of the Poisson model are derived by equating the derivative of the likelihood function to zero. $(\hat{\underline{\boldsymbol{\beta}}})$ which represent the values of the parameter vector $(\underline{\boldsymbol{\beta}})$ that must satisfy the following system of equations:

$$\partial_n(\hat{\beta}_j, y, x) = 0 \quad \dots\dots\dots..(6)$$

We note that Equation (5), which represents the maximum likelihood estimator vector ($\widehat{\boldsymbol{\beta}}$)for the Poisson regression model, is nonlinear in the parameters. the log-likelihood function of the model is a concave function; therefore, these equations can be solved using iterative methods. The) Newton–Raphson**)** method is considered the most suitable for solving concave functions. This method is based on providing initial estimated values for the model parameters ($\widehat{\boldsymbol{\beta}}_{\mathbf{0}}$) through which a second-order approximation of the log-likelihood function $\boldsymbol{L}(\boldsymbol{\beta})$ can be obtained, as follows: [5].

$$L^*(\beta) = L(\hat{\beta}_0) + \partial_n(\hat{\beta}_0)'(\beta - \hat{\beta}_0) + \frac{1}{2}(\beta - \hat{\beta}_0)'H_n(\hat{\beta}_0)(\beta - \hat{\beta}_0) \approx L(\beta) \quad \ldots\ldots..(7)$$

Then, we maximize $\boldsymbol{L}^*(\boldsymbol{\beta})$ with respect to the parameter$(\boldsymbol{\beta})$ to obtain a new estimate of the parameters, denoted by $(\widehat{\boldsymbol{\beta}}')$ **.** The condition for the maximization process is given as follows:z

$$\partial_n(\hat{\beta}_0) + H_n(\hat{\beta}^0)(\hat{\beta}' - \hat{\beta}^0) = 0$$

$$\partial_n(\hat{\beta}_0) + (H_n(\hat{\beta}^0)\hat{\beta}' - H_n(\hat{\beta}^0)\hat{\beta}^0) = 0$$

$$\hat{\beta}' = \hat{\beta}^0 - [H_n(\hat{\beta}^0)]^{-1}\partial_n(\hat{\beta}_0) \quad \ldots\ldots\ldots..(8)$$

We stop iterating the Newton–Raphson procedure and obtaining the Poisson regression model estimators when the difference between ($\hat{\beta}^{t+1}$ **و** $\hat{\beta}^{t}$) is less than 10^{-5}**..**

Properties of The Maximum Likelihood Estimators

The response variable (Y) in the Poisson regression model is a nonlinear function of the parameter vector; therefore, the sampling distribution of the maximum likelihood estimators does not possess large-sample properties. As a result, these estimators have certain properties, the most important of which are:

1- comparatively biased
2- approximated from normal
3- comparatively efficient

Least Absolute Deviation Method (LAD)

The Least Absolute Deviation method (LAD) is one of the less commonly used techniques for estimating the parameters of linear regression compared to other methods. It is somewhat similar to the Ordinary Least Squares (OLS) method**.** the difference between them is that OLS aims to minimize the sum of squared random errors, whereas the LAD method aims to minimize the sum of absolute random errors, as follows:**[6] [7] [8]**

$$f(\beta) = \|y - \beta x\|$$

$$f(\beta) = \sum_{i=1}^{n}\left|y_i - \sum_{j=1}^{m}\beta_j x_{ij}\right| \quad \ldots\ldots(9)$$

Assume that there is a function g(z) defined on the absolute value of (Z), such that: $\mathbf{g}(\mathbf{z}) = |\mathbf{z}|$
where g(z) is a differentiable function defined for all values except at (Z = 0), as follows:

$$g'(z) = \frac{z}{|z|}$$

By substituting each (z) in the above expression with $f(\beta)$, and replacing (|z|) with ($|f(\beta)|$, and then taking the derivative with respect to the unknown parameters (β_r) and setting the derivative equal to zero, we obtain:

$$\frac{\partial f(\beta)}{\partial \beta_r} = \sum_{i=1}^{n} \frac{y_i - \sum_{j=1}^{m} \beta_j x_{ij}}{\left|y_i - \sum_{j=1}^{m} \beta_j x_{ij}\right|} (-x_{ir}) = 0 \qquad \dots\dots(10)$$

r=1,2,…….m

$$\sum_{i=1}^{n} \frac{\beta_j x_{ij}}{U_i} = \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{\beta_j x_{ir} x_{ij}}{U_i} \qquad \dots\dots(11)$$

Let (W) be a diagonal matrix such that **:**

$$w_{ij} = \frac{1}{|U_i|} \qquad i = j$$

$$w_{ij} = 0 \qquad i \neq j$$

Therefore, the above equations can be written in matrix form as follows:

$$(X'W\,Y) = X'W\,X\,\beta$$

By dividing both sides of the above expression by (X'WX), we obtain:

$$\hat{\beta} = (X'W\,X)^{-1}(X'W\,Y) \qquad \dots\dots(12)$$

In this form, we have obtained the regression parameters using the Least Absolute Deviation (LAD) method.

Comparison criteria

Once the parameters of the Poisson regression model have been estimated,the next step is to select the best method among the approaches used. This is carried out using several selection criteria, including:

1- Coefficient of Determination$(\mathbf{R^2})$

It is used to determine the ability of the linear model to explain the variations occurring in the response variable (y).

The coefficient of determination is used in linear regression models through the analysis of the total sum of squares of deviations, where: [9] [10]

$$\sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n}\left(y_i - \hat{\theta}_i\right)^2 + \sum_{i=1}^{n}\left(\hat{\theta}_i - \bar{y}\right)^2 + 2\sum_{i=1}^{n}\left(y_i - \hat{\theta}_i\right)\left(\hat{\theta}_i - \bar{y}\right) \qquad \dots.(13)$$

$$\sum_{i=1}^{n}(y_i - \bar{y})^2 \text{ refers to the total sum of squared deviations (SST)}$$

$$\sum_{i=1}^{n}\left(y_i - \hat{\theta}_i\right)^2 \text{ refers to the sum of squared residuals (errors)(SSE)}$$

$\sum_{i=1}^{n}\left(y_i - \hat{\theta}_i\right)^2$ refers to the sum of squared explained deviations (SSR)

In the case of estimating the parameters of the linear model using the Ordinary Least Squares (OLS) method, the coefficient of determination can be computed as follows:

$$\mathrm{R}^2 = \frac{\mathrm{SSR}}{\mathrm{SST}} \qquad \dots\dots\dots..(14)$$

Or using the following formula:

$$R^2 = 1 - \frac{SSE}{SST} \qquad \dots\dots\dots..(15)$$

$$SST = SSR + SSE$$

The coefficient of determination has the following properties:

$$0 \leq R^2 \leq 11.$$

2. The value of the coefficient of determination does not decrease when explanatory variables are added to the model.

3. There is a consistency between the value of the coefficient of determination and the significance of the estimated model; the larger the coefficient of determination, the more significant the model, and vice versa.

(MSE) 2- mean square error

The mean squared error (MSE) is considered one of the most important criteria for comparing regression models, and it is calculated using the following formula: **[11] [12]**

$$MSE = \frac{SSE}{n-k} \quad \ldots\ldots\ldots..(16)$$

n : represents the sample size.

k : represents the number of parameters in the model.

$$\boldsymbol{sse} = \sum_{i=1}^{n} {U_i}^2 = \sum_{i=1}^{n} (y - \hat{y})^2 \quad \ldots\ldots\ldots..(17)$$

The SSE can also be calculated using the following formula:

$$sse = sst - ssr$$

$$sst = \sum (Y - \bar{Y})^2 = \sum {y_i}^2 \quad \ldots\ldots\ldots..(18)$$

And SSR is calculated using the following formulas:

$$\text{ssr} = \sum (\hat{Y} - \bar{Y})^2 \quad , \quad \text{ssr} = \hat{b}_1 \sum xy \quad , \quad ssr = {\hat{b}_1}^2 \sum x^2 \quad \ldots\ldots\ldots..(19)$$

Thus, the model with the smaller (MSE) is considered better compared to other models.

Testing the parameters of the Poisson regression model

To test the significance of the estimated parameters of the Poisson model using both methods (Maximum Likelihood method and Least Absolute Deviation method), the Wald test is used. Assuming that the statistical hypothesis of the test is:

$\mathrm{H_0 P}(\theta_0) = 0$

against the alternative hypothesis:

$\mathrm{H_1 P}(\theta_0) \neq 0$

Wald test statistic

To test the significance of the estimated parameters of the Poisson regression model, the Wald test is used. Its general form is given as follows:

$$T_w = \mathrm{P}(\hat{\theta}_u)' \left[\left. \frac{\partial \mathrm{P}(\theta)'}{\partial \theta} \right|_{\hat{\theta}_u} \left[\frac{1}{n} \hat{A}(\hat{\theta}_u)^{-1} \right] \left. \frac{\partial P(\theta)}{\partial \theta'} \right|_{\hat{\theta}_u} \right]^{-1} P(\hat{\theta}_u) \quad \ldots\ldots\ldots..(20)$$

$\hat{\mathrm{A}}(\hat{\theta}_u)^{-1}$ It refers to the inverse of the consistent estimator of the variance matrix (A), which is evaluated at the unrestricted maximum likelihood estimates $(\hat{\theta}_u)$, according to the following expression:

$$A = Lim_{n \to \infty} \frac{1}{n} E \left[\left. \frac{\partial^2 Ln\, f_i}{\partial \theta \partial \theta'} \right|_{\theta_0} \right] \quad \ldots\ldots\ldots..(21)$$

Thus, the value of the Wald test statistic is compared with the tabulated Chi-square value at a significance level of **(** $\frac{\alpha}{2}$ **)** and (n) degrees of freedom. If the value of the Wald statistic is greater than the Chi-square critical value, then the estimated parameters are considered statistically significant. However, if the Wald statistic is less than the Chi-square critical value, then the estimated parameters are considered statistically insignificant.

Result

Applied Aspect

Description of the Research Sample

Road accident data from Dhi Qar Governorate for the year 2018 were used. The dependent variable (Y) represents the number of accidents occurring over time, while the independent variable (X) represents time (in weeks). These accidents are recorded by police stations across the governorate, and the data are then sent to the Police Directorate, specifically the Criminal Statistics Department.

Accidents are classified according to their severity into fatal and non-fatal accidents. They are also categorized based on the type of road: accidents occurring on main roads, secondary roads, highways, and rural roads. Furthermore, road accidents are classified by type: collision accidents (which represent the highest proportion among all types), run-over (pedestrian) accidents, and rollover accidents. There are also other types that are not classified within international standards and indicators for road accidents.

In addition, accidents are categorized according to the driver's compliance with wearing a seat belt: the first category includes drivers who wear a seat belt, the second includes those who do not, and the third includes vehicles not equipped with a seat belt. These and other indicators are unified within official statistical classifications.

The overall number of accidents documented in Dhi Qar Governorate during 2018 reached 804 cases.. The proportion of accidents on main roads was (66.42%), on secondary roads (20.77%), on highways (8.46%), and on rural roads (4.35%). Accordingly, the percentage of deaths resulting from road accidents in the governorate relative to total deaths in Iraq was (10.6%), while the percentage of injuries resulting from road accidents was (8.3%) of the total number of injury cases.

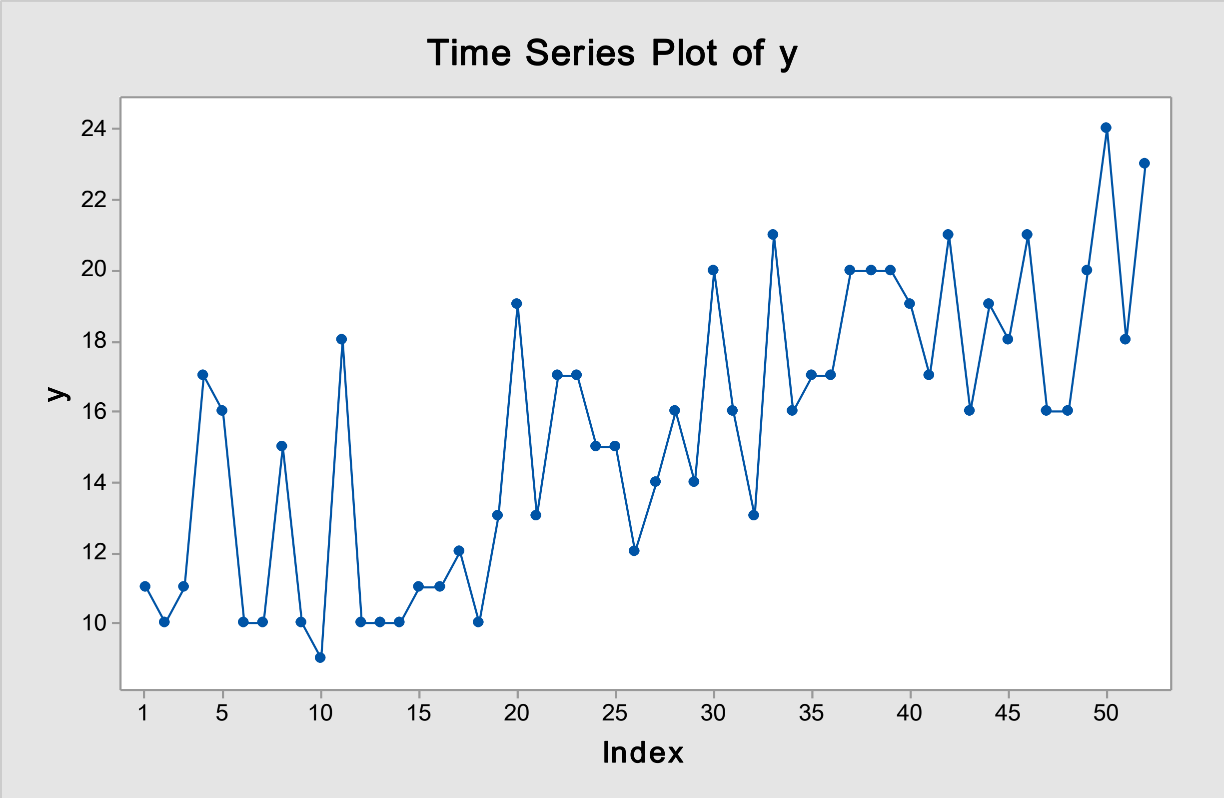

Figure (1)
shows the number of accidents in Dhi Qar Governorate in 2018 over time (in weeks).

The above figure represents the number of road accidents that occurred in Dhi Qar Governorate over time (in weeks) for the period from 1/1/2018 to 30/12/2018. It can be observed that the number of accidents in the final weeks of 2018 is higher than the number of accidents at the beginning of the same year. This is due to several reasons, most notably the coincidence of large gatherings of visitors to Imam Hussein (peace be upon him), coming from the districts and sub-districts of southern Dhi Qar Governorate and Basra Governorate during the last weeks of the year.

Additionally, this period of 2018 coincides with the beginning of the official academic year for schools and universities, leading to increased movement between districts, sub-districts, and the governorate center. Furthermore, the increase in the number of vehicles, the lack of planning in constructing new roads, and the insufficient maintenance of existing roads are all factors contributing to the rise in the number of accidents.

In Figure (1), the horizontal (x) axis represents time in weeks, while the vertical (y) axis represents the number of accidents occurring over time (in weeks). The above graph is based on data collected over a period of (52) weeks, as follows:

Table (1)
shows the number of accidents in Dhi Qar Governorate in 2018 over time (in weeks)

12	11	10	9	8	7	6	5	4	3	2	1	الأسبوع
10	18	9	10	15	10	10	16	17	11	10	11	عـــــــدد الحوادث
24	23	22	21	20	19	18	17	16	15	14	13	الأسبوع

15	17	17	13	19	13	10	12	11	11	10	10	عـــــدد الحوادث
36	35	34	33	32	31	30	29	28	27	26	25	الأسبوع
17	17	16	21	13	16	20	14	16	14	12	15	عـــــدد الحوادث
48	47	46	45	44	43	42	41	40	39	38	37	الأسبوع
16	16	21	18	19	16	21	17	19	20	20	20	عـــــدد الحوادث
								52	51	50	49	الأسبوع
								23	18	24	20	عـــــدد الحوادث

Estimation of the Poisson Regression Equation

For estimating the Poisson regression equation, both the Maximum Likelihood Estimation (MLE) method and the Least Absolute Deviations (LAD) approach are employed, as explained in the theoretical section, this is carried out as follows:

1- Maximum Likelihood Estimation Method (MLE)

The Maximum Likelihood Estimation (MLE) method is considered one of the important parametric methods for estimating the regression equation. Using the statistical software **(**SPSS**)** and **(**Minitab**)**, the Poisson regression equation was estimated as follows:

$$\ln(\hat{y}) = 2.3550 + 0.0132x_i$$

The following figure shows the dispersion of random errors around the regression line estimated using the (MLE) method.

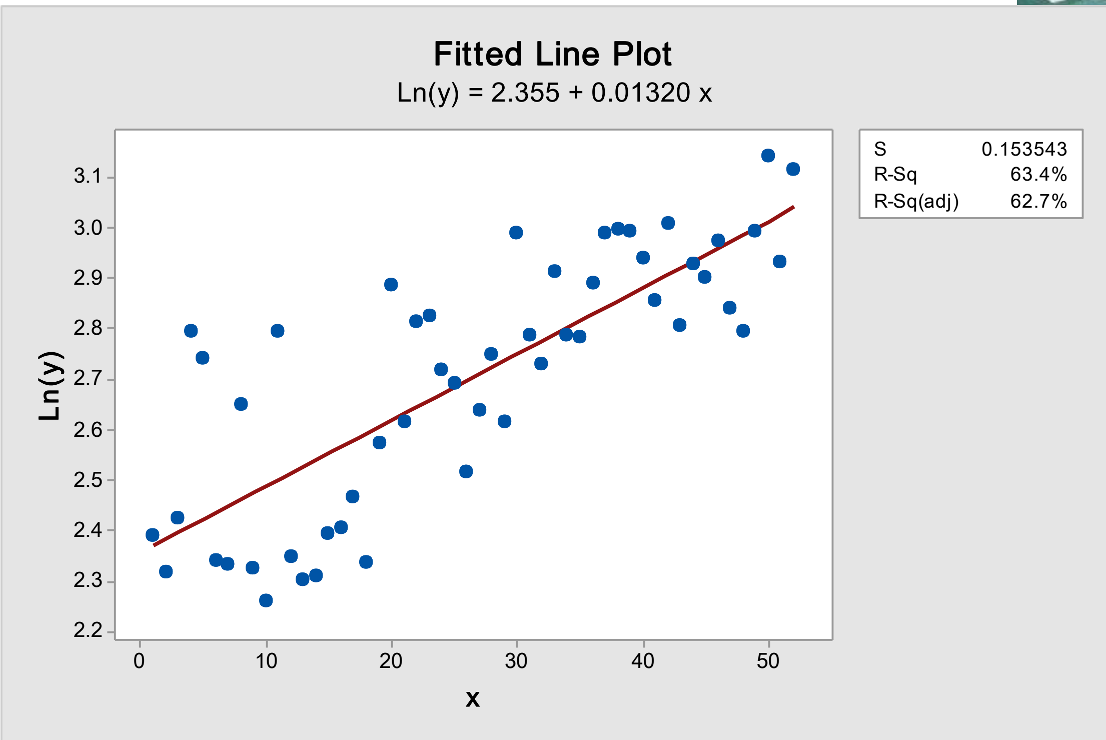

Figure (2)
shows the extent to which the regression line fits the random errors according to the (MLE) method.

The above figure shows the extent to which the random errors deviate from the estimated Poisson regression equation at some points, and their closeness to the regression line at other points. Points that lie above the regression line indicate positive random errors, while points that lie below the regression line indicate negative random errors. As for the points that lie on the regression line, the error value at those points is equal to zero.

2- Least Absolute Deviations Method (LAD)

The Least Absolute Deviations (LAD) method is considered one of the parametric methods, and it is similar to the Least Squares method used in estimating the regression equation. The Poisson regression equation was estimated as follows:

$$\ln(\hat{y}) = 2.320 + 0.01216x_i$$

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P-ISSN:3006-0265 | E-ISSN:3007-4681
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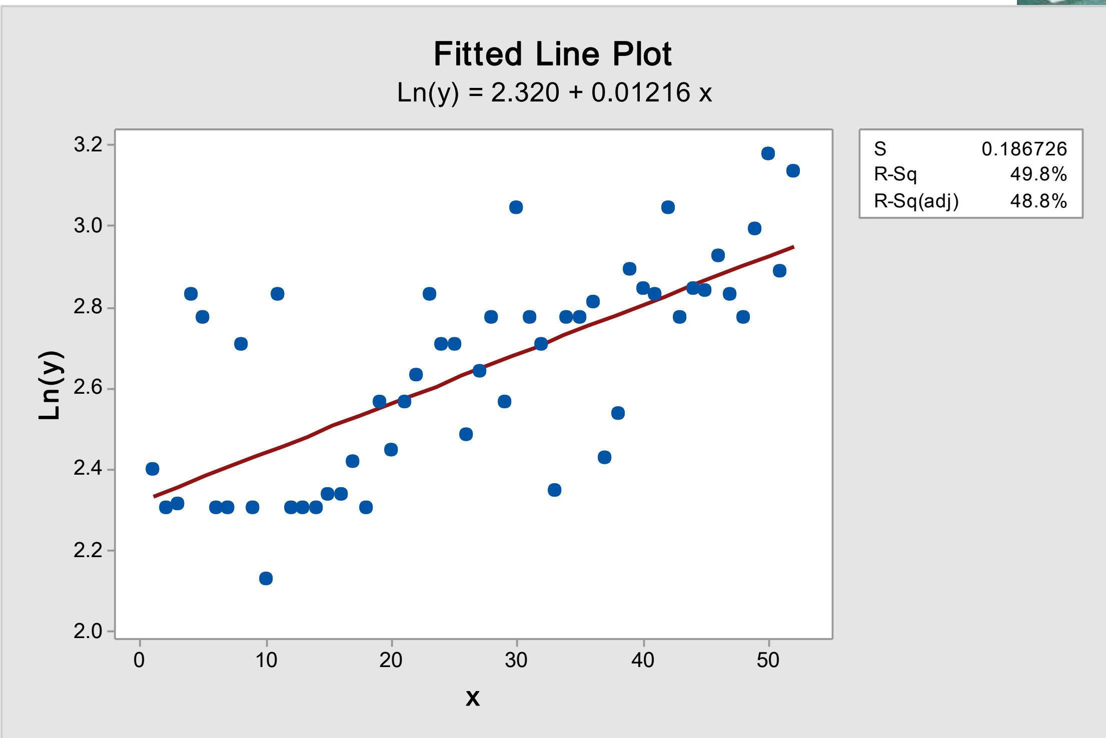

Figure (3)
shows the extent to which the regression line fits the random errors according to the (LAD) method.

The above figure shows the extent to which the random errors deviate from the estimated Poisson regression equation using the (LAD) method at some points, and their closeness to the regression line at other points. It is observed that the random errors are more widely dispersed from the regression line using this method compared to the (MLE) method. Points that lie above the regression line indicate positive random errors, while points that lie below the regression line indicate negative random errors. As for the points that lie on the regression line, the error value at those points is equal to zero.

In general, the results obtained for the parameters of the Poisson regression model for the two aforementioned methods, using the statistical software **(**SPSS and Minitab**)**, can be summarized in the following table:

Table (2)
shows the estimation of the parameters of the Poisson model using the Maximum Likelihood Estimation (MLE) method and the Least Absolute Deviations (LAD) method.

Estimation Method	Parameters	Parameter Estimation
MLE	β_0	**2.3550**
	β_1	**0.0132**
LAD	β_0	**2.320**
	β_1	**0.01216**

Significance Test of the Estimated Parameters

For the purpose of testing the statistical significance of the estimated parameters **(**β_0 , β_1) for the models estimated using the Maximum Likelihood Method and the Least Absolute Deviations Method, the **(**Wald**)** statistic can be employed to assess the significance of these parameters. This is done by comparing the value of the Wald statistic with the tabulated Chi-square value at a significance level of **(** $\frac{0.05}{2}$ **)** and degrees of freedom (n = 52).

If the calculated value of the **(**Wald**)** statistic is greater than the tabulated Chi-square value, the estimated parameter is considered statistically significant. Conversely, if the calculated Wald statistic is less than the tabulated Chi-square value, the estimated parameter is not statistically significant.

Furthermore, the significance of the estimated parameters can also be determined by comparing the P-value with the significance level (0.05). If the P-value is less than (0.05), this indicates that the estimated parameters are statistically significant; otherwise, if the P-value is greater than (0.05), the estimated parameters are not statistically significant, as shown in the following table**:**

Table (3)
Significance of the Estimated Parameters According to the Wald Test

Estimation Method	Parameters	Parameter Estimation	Wald	P-Value
MLE	β_0	**2.3550**	**35.220**	**0.000**
	β_1	**0.0132**	**38.145**	**0.000**
LAD	β_0	**2.320**	**34.764**	**0.000**
	β_1	**0.01216**	**36.876**	**0.000**

From the results presented in Table (3), it is observed that the estimated parameters **(**β_0 , β_1) are statistically significant for both methods. This is determined by comparing the value of the Wald statistic with the tabulated Chi-square value (33.963), where all calculated Wald statistics for the estimated parameters are greater than the tabulated Chi-square value. In addition, the P-values are less than the significance level (0.05),

Comparison between the (MLE) and the (LAD) Method

For the purpose of selecting the best method for estimating the Poisson regression model, the following criteria were used:

1- Coefficient of Determination (R^2)

The model with the higher value of (R^2) is considered better compared to other models. The coefficient of determination indicates the extent to which the regression equation explains the variations in the predicted value **(**$\hat{y}$ **)** resulting from changes in the explanatory variable (X). It was found to be (63.4%) for the model estimated using **(**MLE**)** Method, and (49.8%) for the model estimated using the (LAD) Method.

2- Mean Squared Error (MSE)

The model with the lower Mean Squared Error (MSE) is considered better than other models. The **(**MSE**)** value was (0.02358) for the model estimated using the Maximum Likelihood Estimation (MLE) method, and (0.03487) for the model estimated using the Least Absolute Deviations (LAD) method, as shown in the following table:

Table (5)
Comparison between the Estimated Models According to the Criteria (R^2 ، MSE^2)

Estimation Method	Parameter Estimation		MSE	R^2
	β_0	β_1		
MLE	**2.3550**	**0.0132**	**0.02358**	63.4**%**
LAD	**2.320**	**0.01216**	**0.03487**	49.8**%**

From the results in Table (5), it is observed that the Mean Squared Error (MSE) for the Maximum Likelihood Estimation (MLE) method is (0.02358), while it is (0.03487) for the Least Absolute Deviations (LAD) method. In addition, the **(**MLE**)** method achieved the highest coefficient of determination (R^2), which equals (63.4%). Therefore, the Maximum Likelihood Estimation (MLE) method is considered better than the **(**LAD**)** method.

Conclusions

1- Based on the criteria used for comparing the estimated models, it was found that the best method for estimating the Poisson model is the Maximum Likelihood Estimation method (MLE) .

2- The study data showed that the number of road accidents in Dhi Qar Governorate during the last weeks of 2018 was higher than in the other weeks. This is attributed to the start of the official school and university semesters, as well as the coincidence of these weeks with the Arabian pilgrimage of Imam Hussein, which led to a large gathering of visitors in the governorate.

3- The study of road accidents that occurred in the governorate during 2018 showed that collision accidents ranked first, pedestrian-vehicle (run-over) accidents ranked second, and rollover accidents came last.

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